Temperature Turbulence Spectrum for High-Temperature Radiating Gases

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The influence of radiation on thermal turbulence spectra is studied theoretically in the case of homogeneous and isotropic turbulence in high-temperature radiating gases. A statistical narrow-band model is used to compute radiative properties of real gases such as CO_2 and H_2O, and an Onsager-type model is used as closure for the temperature variance spectrum equation. It is found that radiation acts as a dissipative process with a coefficient $N(k)$ smaller than the conductive one ak^2 for small eddies and greater than ak^2 for large eddies. The critical wave number, for which conductive and radiative processes are of the same order of magnitude, is found to be close to $k = 100\ m^{-1}$ for pure H_2O or pure CO_2 in the temperature range [400 K, 2000 K]. At high temperature and small values of the viscous dissipation rate of turbulent kinetic energy, ε, radiation significantly modifies the temperature variance spectrum in the region typically between $10^{-3}\ k_d$ and k_d, where k_d is the Kolmogorov wave number. The effects of radiation increase with temperature but decrease with ε.

Nomenclature

a	=	thermal diffusivity
C_p	=	specific heat at constant pressure
E_T	=	temperature variance spectral density
$F(k)$	=	spectral flux at wave number k
I_ν	=	radiation spectral intensity
I_ν^b	=	blackbody spectral intensity
$\bar{k}$	=	mean line intensity to mean line spacing ratio inside $\Delta\nu$
$\mathbf{k}$	=	wave vector
$N(k)$	=	radiative dissipation term
$P(S)$	=	probability distribution law of line intensities
p	=	pressure
$\mathbf{q}_r$	=	radiative flux
r	=	column length
$\mathbf{r}$	=	spatial position
S	=	radiative source term
T	=	mean temperature
T_T	=	transfer term in the spectral equation
t	=	time
$\mathbf{u}$	=	propagation direction
u_j	=	velocity component
x	=	molar fraction of the absorbing species
x_j	=	$\mathbf{r}$ coordinate
$\bar{\gamma}$	=	mean line half-width inside $\Delta\nu$
$\Delta\nu$	=	narrow-band model spectral range
$\bar{\delta}$	=	mean line spacing inside $\Delta\nu$
δI_ν	=	radiation intensity fluctuation
δI_ν^b	=	blackbody intensity fluctuation
ε	=	turbulent kinetic energy dissipation rate
θ	=	temperature fluctuation
κ_ν	=	spectral absorption coefficient
ν	=	kinematic viscosity
ρ	=	density
τ_c	=	conductive characteristic time
τ_e	=	inertial time scale
τ_r	=	radiative characteristic time
τ_ν	=	spectral transmissivity
χ	=	temperature variance dissipation rate
χ_R	=	total radiative dissipation rate

Subscripts

ν	=	spectral quantity

Symbols

$u \otimes v$	=	convolution product
$\hat{\Phi}$	=	three-dimensional Fourier transform
$\bar{\Phi}$	=	quantity Φ averaged over $\Delta\nu$

Introduction

KNOWLEDGE of thermal turbulence spectra is often required for better understanding and modeling of turbulent flows (second-order closures for example, Newman et al.[1]) and for the study of acoustical or electromagnetic wave propagation through the turbulent medium. However, high-temperature systems (combustion applications) and even low-temperature media (atmospheric applications) are usually concerned with radiating gases such as CO_2 and H_2O, and radiative transfer in these systems acts as a dissipative process, especially for large-sized structures for which the optical thickness becomes important. This fact was first identified in the pioneer studies by Townsend[2] and Spiegel,[3] and then applied to atmospheric studies in more recent papers (Shved[4] and Coantic and Simonin[5]). It was found that gas and particle (water droplet) radiation may greatly modify the structure of temperature fluctuation spectra in the planetary boundary layer.

For high-temperature radiating gases, great attention was given in the literature to the influence of turbulence on radiation since this problem is important in the simulation of radiation from turbulent flames. Both theoretical and experimental studies have shown that the emitted radiation from the flame may be significantly increased as a result of temperature fluctuations (Kabashnikov and Kmit[6]; Jeng et al.[7]; Faeth et al., 1985[8]; Faeth, 1986[9]; Gore et al.[10,11]; Song and Viskanta[12]; Kounalakis et al.[13]). But to our knowledge, no attention was given to the effects of radiation on thermal turbulence in high-temperature media.

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The aim of this paper is to investigate the effects of radiation on temperature turbulence spectra for real gases such as CO_2 and H_2O, at different temperature levels, but in the simple case of homogeneous and isotropic turbulence. The analysis is carried on for small temperature fluctuations in such a way that blackbody intensity may be linearized around it's value for the mean temperature. The model used for molecular gas radiation is briefly described in the next section, and the resulting radiative dissipation coefficients are discussed and compared with conductive coefficients in the following one. The closure model for the dynamic equation of thermal turbulence spectra is then described, and the results for radiating gases are discussed in the last section.

Radiative Model

Absorption spectra of gases such as CO_2 and H_2O include several thousand significant lines and a line-by-line (LBL) calculation is impracticable in radiative transfer calculations. Furthermore, a mean absorption coefficient is not physically meaningful since it ignores spectral line correlation phenomena. We use here a statistical narrow-band model to compute the transmissivity $\bar{\tau}_\nu$ of a gaseous column, averaged over an electromagnetic spectral range $\Delta\nu$

$$\bar{\tau}_\nu = \frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-\kappa_\nu r)\mathrm{d}\nu, \tag{1}$$

where κ_ν is the spectral absorption coefficient and r the column length. The lines inside $\Delta\nu$ are assumed to be randomly placed and their intensities S follow the two parameter (R and S_M) probability law

$$P(S) = \frac{1}{S \operatorname{Log} R}\left[\exp\left(-\frac{S}{S_M}\right) - \exp\left(-\frac{RS}{S_M}\right)\right] \tag{2}$$

introduced by Malkmus.[14] The free parameters S_M and R may be interpreted as the maximum line intensity and the maximum to minimum line intensity ratio, respectively.[14,15] These assumptions lead to the following expression for the averaged transmissivity over $\Delta\nu$[15]

$$\bar{\tau}_\nu = \exp\left[-\frac{2\bar{\gamma}}{\bar{\delta}}\left(\sqrt{1+\frac{\bar{k}\bar{\delta}pxr}{\bar{\gamma}}} - 1\right)\right] \tag{3}$$

where p and x are the pressure and the molar fraction of the absorbing species, that are assumed to be constant. The model parameters $\bar{k}$, $\bar{\gamma}$, and $\bar{\delta}$ designate, respectively, mean line intensity to mean line spacing ratio, mean line half-width at half-maximum, and the weighted mean line spacing inside $\Delta\nu$

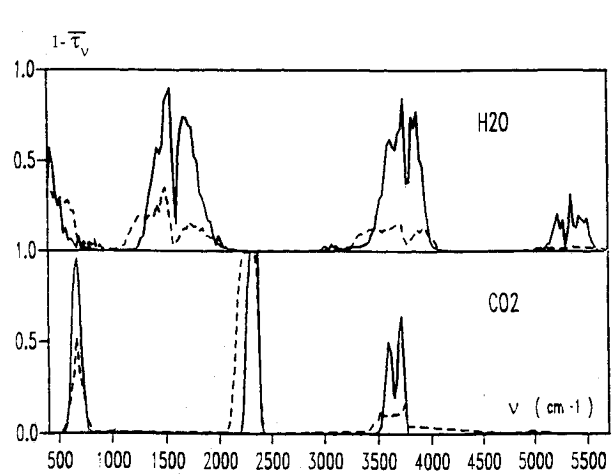

Fig. 1 H_2O and CO_2 absorption spectra for 5 cm column length at ——— 400 K and - - - 2000 K.

(see Ref. 15, for example). These parameters have been generated from LBL calculations in previous studies (Hartmann et al.[16] and Soufiani et al.[17]) where this radiative model was found to give the best agreement with LBL. The statistical narrow-band model is used here with a spectral resolution $\Delta\nu = 25\ \mathrm{cm}^{-1}$, and for the absorption bands of CO_2 and H_2O centered between the wave numbers $\nu = 150\ \mathrm{cm}^{-1}$ and $\nu = 6000\ \mathrm{cm}^{-1}$. Figure 1 shows the computed spectra for two temperature conditions, that will be considered in the following. The radiative transfer equation may be written for a nonscattering medium as

$$\bar{I}_\nu(\mathbf{r}, \mathbf{u}) = \bar{I}_\nu(\mathbf{0}, \mathbf{u})\bar{\tau}_\nu(0, r) + \int_0^r \overline{I^b_\nu}(s)\frac{\partial\bar{\tau}_\nu}{\partial s}(s, r)\mathrm{d}s \tag{4}$$

or in a differential form and before averaging over $\Delta\nu$ as

$$\mathbf{u}.\nabla I_\nu(\mathbf{r}, \mathbf{u}) = \tau'_\nu(0)[I_\nu(\mathbf{r}, \mathbf{u}) - I^b_\nu(\mathbf{r})] \tag{5}$$

where $\mathbf{u}$ is the unit vector in the propagation direction, I^b_ν is the blackbody intensity, and $\tau'_\nu(0) = \partial\tau_\nu/\partial r$ ($r = 0$) stands for $-\kappa_\nu$.

Equation for the Temperature Spectrum of Radiating Gases

The assumptions made in the following analysis are that:

a) The turbulent field is homogeneous and isotropic.

b) The fluctuating part θ of the temperature is small compared to the mean temperature T. This enables a linear decomposition of I^b_ν with θ. The effects of Planck's function nonlinearities are discussed in the appendix. It is found that in the worst case, nonlinear terms remain of the same order of magnitude as θ/T.

c) As a result of the previous assumption, thermophysical and radiative properties are assumed to be constant and do not depend on θ. Nevertheless, these properties depend on the mean temperature and are computed from T.

With these assumptions, we use a formalism similar to that of Coantic and Simonin[5] to obtain the equation for the temperature spectrum. Temperature fluctuation θ obeys the energy conservation equation

$$\frac{\partial\theta}{\partial t} + u_j\frac{\partial\theta}{\partial x_j} = a\frac{\partial^2\theta}{\partial x_j^2} + S \tag{6}$$

where a is the molecular thermal diffusivity and S the radiative source term related to the divergence of the radiative flux $\mathbf{q}_r$

$$S = -\frac{1}{\rho C_p}\nabla.\mathbf{q}_r \tag{7}$$

The three-dimensional Fourier transform of Eq. (6) leads to

$$\frac{\partial\hat{\theta}}{\partial t}(\mathbf{k}, t) + ik_j\hat{u}_j\otimes\hat{\theta}(\mathbf{k}, t) = -ak^2\hat{\theta}(\mathbf{k}, t) - N(k)\hat{\theta}(\mathbf{k}, t) \tag{8}$$

where $\mathbf{k}$ is the turbulence wave vector and $N(k)$ is a coefficient obtained from the linearization of the radiative field

$$\hat{S}(\mathbf{k}, t) = -N(k)\hat{\theta}(\mathbf{k}, t) \tag{9}$$

or

$$N(k) = \int_0^\infty N_\nu(k)\mathrm{d}\nu \tag{10}$$

with

$$N_\nu(k) = \frac{1}{\rho C_p} \frac{\widehat{\nabla . \mathbf{q}_{r\nu}}(\mathbf{k}, t)}{\hat{\theta}(\mathbf{k}, t)} \tag{11}$$

The expression for $N_\nu(k)$ is derived from the radiative transfer equation after the decomposition of the intensity I_ν into its mean and fluctuating components

$$I_\nu(\mathbf{r}, \mathbf{u}) = I_\nu^b(T) + \delta I_\nu(\mathbf{r}, \mathbf{u}) \tag{12}$$

$$I_\nu^b(\mathbf{r}) = I_\nu^b(T) + \delta I_\nu^b(\mathbf{r}) \tag{13}$$

The analysis is carried out first for the real spectral intensity I_ν (before averaging over $\Delta\nu$) until a linear expression of $N_\nu(k)$ vs τ_ν is obtained. This expression is then averaged over $\Delta\nu$ to enable the use of the statistical narrow-band model. The radiative transfer equation may be written in terms of δI_ν

$$\mathbf{u} . \nabla \delta I_\nu(\mathbf{r}, \mathbf{u}) = \tau_\nu'(0)[\delta I_\nu(\mathbf{r}, \mathbf{u}) - \delta I_\nu^b(\mathbf{r})] \tag{14}$$

which leads after Fourier transformation to

$$\widehat{\delta I_\nu}(\mathbf{k}, \mathbf{u}) = \frac{\tau_\nu'(0)\widehat{\delta I_\nu^b}(\mathbf{k})}{\tau_\nu'(0) - i\mathbf{k}.\mathbf{u}} \tag{15}$$

The divergence of the spectral radiative flux is obtained by an integration over the propagation direction

$$\nabla . \mathbf{q}_{r\nu}(\mathbf{r}) = \int_{4\pi \mathrm{str}} \mathbf{u} . \nabla \delta I_\nu(\mathbf{r}, \mathbf{u}) \mathrm{d}\Omega \tag{16}$$

and it's Fourier transform is

$$\widehat{\nabla . \mathbf{q}_{r\nu}}(\mathbf{k}) = \int_{4\pi \mathrm{str}} i\mathbf{k}.\mathbf{u}\widehat{\delta I_\nu}(\mathbf{k}, \mathbf{u}) \mathrm{d}\Omega \tag{17}$$

or from Eq. (15):

$$\widehat{\nabla . \mathbf{q}_{r\nu}}(\mathbf{k}) = \int_{4\pi \mathrm{str}} i\mathbf{k}.\mathbf{u} \frac{\tau_\nu'(0)\widehat{\delta I_\nu^b}(\mathbf{k})}{\tau_\nu'(0) - i\mathbf{k}.\mathbf{u}} \mathrm{d}\Omega \tag{18}$$

This integration can be handled analytically by setting $\mu = (\mathbf{k}.\mathbf{u})/k$

$$\widehat{\nabla . \mathbf{q}_{r\nu}}(\mathbf{k}) = 2\pi i k \widehat{\delta I_\nu^b}(\mathbf{k}) \int_{-1}^{1} \frac{\mu \mathrm{d}\mu}{1 - i\mu k/\tau_\nu'(0)} \tag{19}$$

which leads, after a separation of the integrand into real and imaginary parts, to:

$$\widehat{\nabla . \mathbf{q}_{r\nu}}(\mathbf{k}) = -4\pi \widehat{\delta I_\nu^b}(\mathbf{k}) \tau_\nu'(0) \left[1 - \frac{\tau_\nu'(0)}{k} \arctan\left(\frac{k}{\tau_\nu'(0)}\right)\right] \tag{20}$$

This expression is still nonlinear with respect to τ_ν, but it can be changed to

$$\widehat{\nabla . \mathbf{q}_{r\nu}}(\mathbf{k}) = -4\pi \widehat{\delta I_\nu^b}(\mathbf{k}) \left(\tau_\nu'(0) + \int_0^\infty \frac{\partial^2 \tau_\nu}{\partial r^2}(r) \frac{\sin(kr)}{kr} \mathrm{d}r\right) \tag{21}$$

since we have the equalities

$$\frac{a}{b} \arctan\left(\frac{b}{a}\right) = a \int_0^\infty e^{-ar} \frac{\sin(br)}{br} \mathrm{d}r$$

and

$$\tau_\nu'^2(0)\, e^{\tau_\nu'(0)r} = \kappa_\nu^2 e^{-\kappa_\nu r} = \frac{\partial^2 \tau_\nu}{\partial r^2}(r)$$

Equations (11) and (21), together with the linearization of blackbody intensity

$$\delta I_\nu^b(\mathbf{r}) = \frac{\partial I_\nu^b}{\partial T}(T)\theta(\mathbf{r})$$

or

$$\widehat{\delta I_\nu^b}(\mathbf{k}) = \frac{\partial I_\nu^b}{\partial T}(T)\hat{\theta}(\mathbf{k}) \tag{22}$$

lead finally to the expression of $N_\nu(k)$

$$N_\nu(k) = -\frac{4\pi}{\rho C_p} \frac{\partial I_\nu^b}{\partial T}(T) \left[\frac{\partial \tau_\nu}{\partial r}(0) + \int_0^\infty \frac{\partial^2 \tau_\nu}{\partial r^2}(r) \frac{\sin(kr)}{kr} \mathrm{d}r\right] \tag{23}$$

which can be averaged over $\Delta\nu$

$$N_{\Delta\nu}(k) = -\frac{4\pi}{\rho C_p} \frac{\partial \overline{I_\nu^b}}{\partial T}(T) \left[\frac{\partial \overline{\tau_\nu}}{\partial r}(0) + \int_0^\infty \frac{\partial^2 \overline{\tau_\nu}}{\partial r^2}(r) \frac{\sin(kr)}{kr} \mathrm{d}r\right] \tag{24}$$

and then integrated over all the electromagnetic spectrum

$$N(k) = \sum_{\mathrm{narrow\,bands}\,\Delta\nu} \Delta\nu N_{\Delta\nu}(k) \tag{25}$$

The equation for the temperature spectrum is then obtained classically from Eq. (8). It is written

$$\frac{\partial E_T}{\partial t}(k, t) = T_T(k, t) - 2(ak^2 + N(k))E_T(k, t) \tag{26}$$

where $E_T(k, t)\,\mathrm{d}k$ is the contribution of the turbulence spectral range $[k, k + \mathrm{d}k]$ to the temperature fluctuation variance ($\int_0^\infty E_T(k, t)\,\mathrm{d}k = \langle\theta^2\rangle$), $T_T(k, t)$ is the transfer term related to the convection of temperature fluctuations by the turbulent flow.

It is seen from Eq. (26) that radiation acts as a dissipative process, just like thermal molecular conduction expressed by the term ak^2. The radiative dissipation term $N(k)$ is compared in Figs. 2 and 3 to the conductive term for pure H_2O and CO_2, respectively, at different temperatures. It is seen that $N(k)$ is preponderant for small wave numbers (e.g., large length scales and then optically thick media) while the conductive dissipation term becomes predominant for high values of k, which correspond to small eddies and then optically thin media. The asymptotic variations of $N(k)$ in the limit $k \to 0$ are of the kind $N(k) \sim k^2$, since radiation acts like molecular conduction in the Rosseland limit. On the other hand, for $k \to \infty$, $N(k)$ tends to a constant value. The intersection between $N(k)$ and ak^2 curves is located near $k = 100\ \mathrm{m}^{-1}$. This value is mainly imposed by the medium opacity and thermal diffusivity. It varies only slightly with temperature (from 90 m^{-1} to 110 m^{-1} for H_2O, and from 30 m^{-1} at 400 K to 200 m^{-1} at 2000 K, for CO_2) since both the derivative $\partial I_\nu^b/\partial T$ and thermal diffusivity increase with temperature.

From another point of view, $N(k)$ and ak^2 may be interpreted as the inverse of radiative and conductive characteristic times, respectively: $\tau_r(k) = 1/N(k)$ and $\tau_c(k) = 1/ak^2$. These two time scales may be compared to the inertial time scale

$$\tau_e(k) = (\varepsilon^{1/3}k^{2/3})^{-1}$$

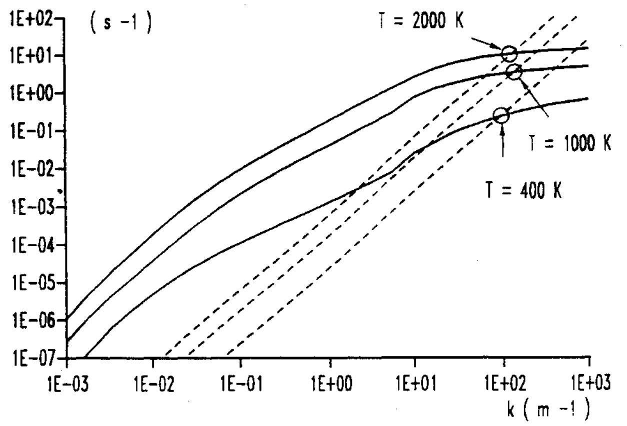

Fig. 2 ——— radiative ($N(k)$) and - - - conductive (ak^2) dissipation terms for turbulent temperature spectrum. H_2O at p = 1 atm and different temperatures.

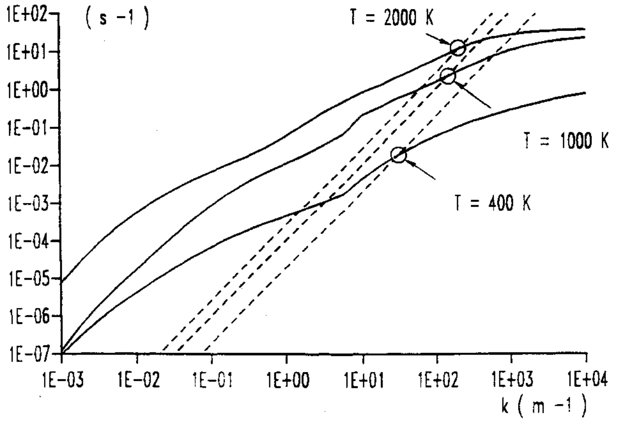

Fig. 3 ——— radiative ($N(k)$) and - - - conductive (ak^2) dissipation terms for turbulent temperature spectrum. CO_2 at p = 1 atm and different temperatures.

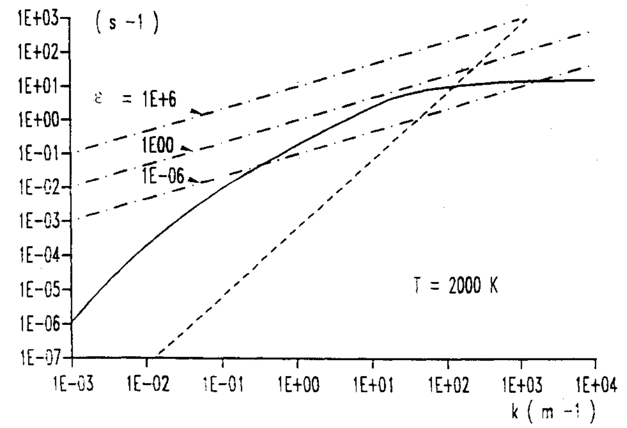

Fig. 4 Inverse of the characteristic times ——— τ_r, - - - τ_c, and - · - · τ_e for water vapor at 2000 K.

where ε is the rate of viscous dissipation of turbulent kinetic energy. The intersection between τ_c and τ_e curves occurs at the Corrsin wave number $k_c = (\varepsilon/a^3)^{1/4}$, which is close to the Kolmogorov wave number $k_d = (\varepsilon/\nu^3)^{1/4}$ for gases (Prandtl number ≈ 1). The curves τ_e and τ_r may or may not intersect. It has been shown by Schertzer and Simonin[18] that when τ_e and τ_r curves intersect, there is an "inertial-radiative" subrange where τ_r is the smallest characteristic time. Inside this subrange, the temperature spectrum slope was found close to -3. Figure 4 shows that for pure H_2O at 2000 K, such subrange should exist for $\varepsilon < 10^{-1}$ m^2 s^{-3}. This critical ε value decreases with temperature. For CO_2 at 2000 K, the critical ε is about 10^{-2} m^2 s^{-3}.

Closure Model

The dynamic equation for E_T is solved in this study by using an Onsager-type closure model for the transfer term $T_T(k, t)$. The spectral flux function $F(k)$ is written in the form

$$F(k) = -\int_0^k T_T(k')\mathrm{d}k' = s(k)E_T(k) \tag{27}$$

where the proportionality function $s(k)$ depends only on the dynamical conditions and is assumed to be unmodified by radiation. The Hill's generalization of the Corrsin-Pao model to the case of a passive scalar[19] is used. It leads to

$$s(k) = \frac{\beta^{-1}k}{\tau_e + \tau_2} \tag{28}$$

where β is the Corrsin-Obukhov constant, τ_e is the inertial time scale and τ_2 is another characteristic time defined by

$$\tau_2 = Q\sqrt{\frac{\nu}{\varepsilon}} \tag{29}$$

where the parameter Q is a new model constant. We use the values $\beta = 0.767$ and $Q = 2.2$ as determined by Hill[19] from the comparison with the experimental data of Williams and Paulson.[20] This closure is used for radiating gases without any modification since the model is assumed to be valid for any Prandtl number,[19] and then, does not depend on the physical process of temperature fluctuation dissipation.

Replacement of Eq. (27) into Eq. (26) leads, in the stationary case, to the explicit solution for $E_T(k)$

$$E_T(k) = \frac{\lambda}{s(k)} \exp\left(-2\int_0^k \frac{ak'^2 + N(k')}{s(k')}\mathrm{d}k'\right) \tag{30}$$

where λ is an integration constant, calculated by assuming that, in the limit $k \to 0$, the inertio-convective form

$$E_T(k) = \beta\chi\varepsilon^{-1/3}k^{-5/3} \tag{31}$$

is recovered, since dissipation effects are negligible. This behavior leads to $\lambda = \chi$, where χ is the total temperature variance dissipation rate

$$\chi = 2\int_0^\infty (ak^2 + N(k))E_T(k)\mathrm{d}k \tag{32}$$

It has been shown in previous studies that this simple model leads to satisfactory results when compared with experimental data for temperature spectra of nonradiating fluids,[19] and when compared with the results of EDQNM (Eddy-Damped Quasi-Normal Markovian) closure for low-temperature radiating gases, and far from the injection wave number (Coantic and Simonin[5]). This model assumes that the injection of temperature variance is done at $k = 0$, and consequently, cannot predict the spectrum's behavior near the injection wave number for finite turbulence Reynolds number. We have also tested this Onsager-Hill model in the experimental conditions of Williams and Paulson[20] for nonradiating gases. Satisfactory agreement (similar to that in the comparison made by Hill[19]) between predicted and measured one-dimensional temperature spectra is obtained.

Temperature Spectra and Global Radiative Dissipation

For temperature spectrum computation, the main calculation parameters are thermodynamic fluid conditions (tem-

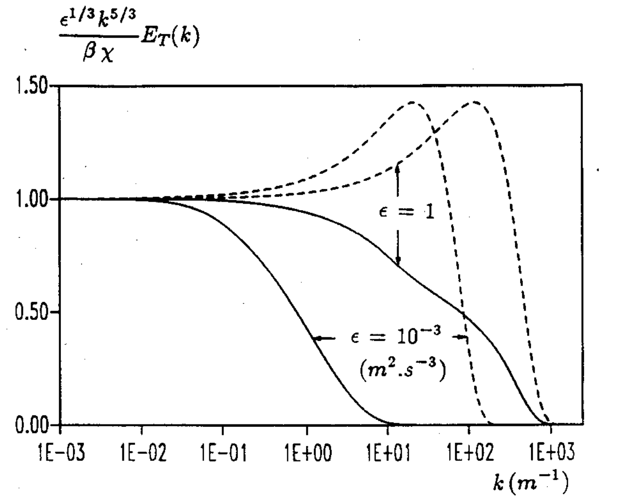

Fig. 5 Compensated thermal turbulence spectra —— with and - - - without radiation for H_2O at $T = 1000$ K and $p = 1$ atm.

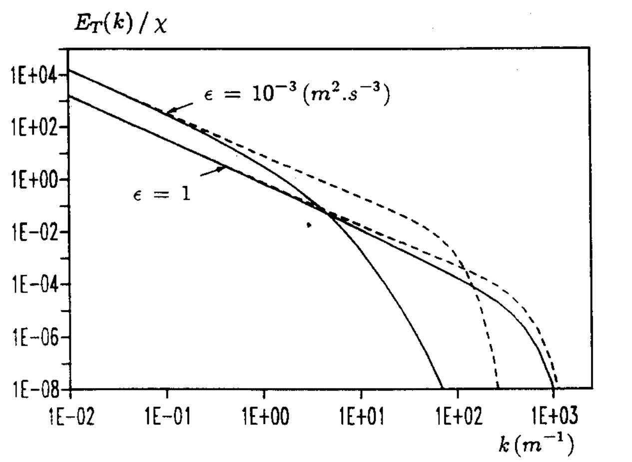

Fig. 6 Noncompensated thermal turbulence spectra in the same conditions as in Fig. 5.

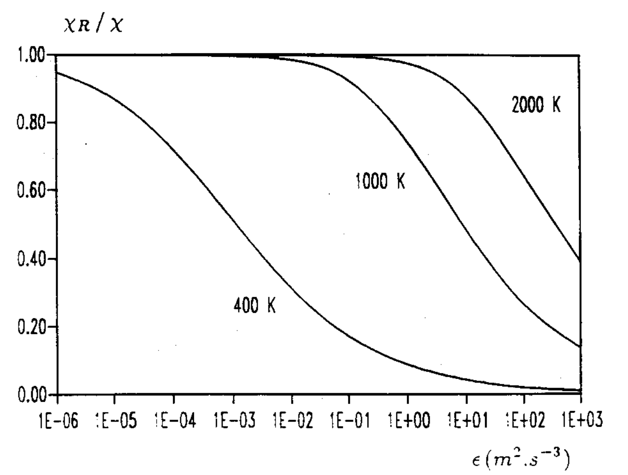

Fig. 7 Radiative to total dissipation ratio versus ε for H_2O at different temperatures.

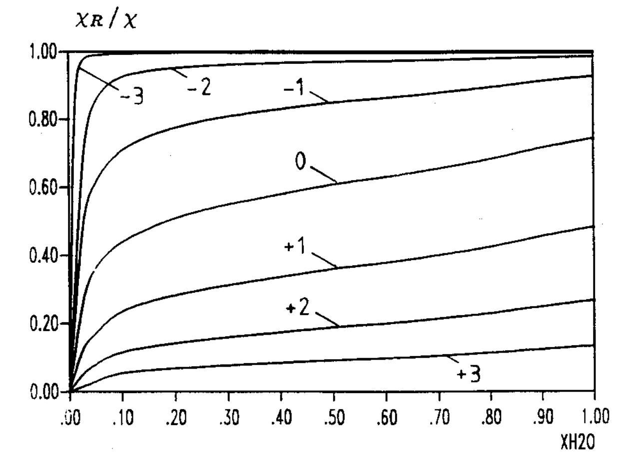

Fig. 8 Radiative to total dissipation ratio for H_2O-air mixtures at 1000 K and for different ε values [$\log_{10}(\varepsilon)$ is the parameter shown in the figure].

perature, pressure and composition), the viscous dissipation rate of turbulent kinetic energy ε, and the total dissipation rate χ, which depends on the injection rate. Once these parameters are specified, the three-dimensional spectrum $E_T(k)$ may be computed from Eqs. (24–25) and Eqs. (28–30).

Figures 5 and 6 show examples of computed spectra for pure water vapor and two ε values in the conditions $T = 1000$ K and $p = 1$ atm. Calculations accounting for radiation and not accounting for radiation are displayed on these figures. The function $E_T(k)$ is normalized in Fig. 5 by using the multiplicative factor $\varepsilon^{1/3}k^{5/3}\beta^{-1}\chi^{-1}$, in order to show clearly the inertio-convective behavior. It is observed that radiation does not affect the small k value region, since spectra are mainly imposed by production and transfer mechanisms in this region. On the other hand, radiation radically modifies the spectral structure typically between $10^{-3}\,k_d$ and k_d, where k_d designates the Kolmogorov wave number. The influence of radiation on thermal turbulence spectra decreases significantly with ε since, for high ε values, the inertial time scale τ_e becomes very small in comparison with the other times and the inertial effects are, therefore, the most important. Qualitatively, the relative effects of radiation may be estimated in a given situation by comparing the global radiative dissipation rate

$$\chi_R = 2\int_0^\infty N(k)E_T(k)\mathrm{d}k \qquad (33)$$

to the total (radiative + conductive) dissipation rate χ given by Eq. (32). Figure 7 displays the ratio χ_R/χ vs ε for pure H_2O at 400 K, 1000 K and 2000 K. The increase in temperature obviously leads to an increase of radiative influence. At 2000 K for instance, the relative dissipation rate caused by radiation is still about 40% for ε as high as 10^3 m^2 s^{-3}, while at 400 K, radiative effects become negligible for ε greater than 10 m^2 s^{-3}.

In the previous discussion, the media considered was pure water vapor, but in practical situations such as in combustion media, molar fractions of radiating gases are generally limited to about 0.2. It is, therefore, interesting to investigate the effects of molar fraction, or, in other words, the effects of medium optical thickness. Figure 8 shows the ratio χ_R/χ for air-H_2O mixtures as a function of x_{H_2O} at $T = 1000$ K and for different values of ε. It is seen that the relative effects of radiation increase first quickly with x_{H_2O} and then slowly and quasilinearly from $x_{H_2O} \approx 0.15$ to $x_{H_2O} = 1$. Radiative effects remain significant for small x_{H_2O} values. In fact, when x_{H_2O} decreases, the radiative dissipation $N(k)$ decreases for small eddies (typically $k > 1$ m^{-1}), but $N(k)$ increases for large sized eddies as a result of the increase in the optical penetration length.

Concluding Remarks

We have shown in this study that radiation may greatly modify the structure of temperature spectra of radiating gases by smoothing the intensity of temperature fluctuations. These effects should be accounted for in the applications requiring the knowledge of the spectral distribution of temperature fluctuations, e.g., wave propagation through radiating gases and the prediction of radiation from flames. The results presented in Figs. 7 and 8 enable a rapid estimation of radiation effects in a given situation. However, when radiation becomes the predominant dissipative process, spectral nonlocal interactions may become important (Schertzer and Simonin[18]); and it would be interesting to compare in this case the results of the Onsager-Hill model with those of more sophisticated models, such as EDQNM closure. This work is actually in progress. Similarly, it should be interesting to investigate the effects of radiation on thermal turbulence spectra in the case of nonisotropic turbulence and the influence of fluctuating molar fraction of the radiating species.

Appendix

We discuss in this appendix the effects of Planck's function nonlinearities with respect to temperature by considering the first neglected term $\theta^2/2\ \partial^2 I_\nu^b/\partial T^2\ (T)$. The error introduced in the physical space by linearizing I_ν^b is of the order

$$\left[\frac{\theta^2}{2}\frac{\partial^2 I_\nu^b}{\partial T^2}(T)\right] \Big/ \left[\theta\frac{\partial I_\nu^b}{\partial T}(T)\right] = \frac{\theta}{T}\left[\frac{\alpha}{2}\frac{e^\alpha + 1}{e^\alpha - 1} - 1\right] \qquad \text{(A1)}$$

with $\alpha = h\nu/kT$. The right-hand side of Eq. (A1) can be written in the form $\theta/T\ g(\nu, T)$ where the function g depends only on the ratio ν/T or on the product λT, where λ is the radiation wavelength. This function is displayed on Fig. A1 vs the dimensionless wavelength $\lambda/\lambda_m(T)$, $\lambda_m(T)$ being the wavelength corresponding to blackbody maximum emission: $\lambda_m(T)\ T = 2898\ \mu$m K. It is seen from this figure that the linearization of I_ν^b is an accurate approximation for the absorption bands located at wavelengths much greater than $\lambda_m(T)$, while nonlinearities are significant for $\lambda < \lambda_m(T)$. Therefore, the accuracy of the linearization depends on the spectral position of the absorption bands for a given molecule at a given temperature. In the worst case, nonlinearity effects remain of the same order of magnitude as θ/T.

In the Fourier space, if we introduce the first neglected term in the decomposition of δI_ν^b, Eq. (22) becomes

$$\widehat{\delta I_\nu^b}(\mathbf{k}) = \frac{\partial I_\nu^b}{\partial T}(T)\hat{\theta}(\mathbf{k}) + \frac{1}{2}\frac{\partial^2 I_\nu^b}{\partial T^2}(T)\hat{\theta}\otimes\hat{\theta}(\mathbf{k}) \qquad \text{(A2)}$$

and the three-dimensional Fourier transform of the energy equation gives

$$\frac{\partial\hat{\theta}}{\partial t}(\mathbf{k}, t) + ik_j\hat{u}_j\otimes\hat{\theta}(\mathbf{k}, t) = -ak^2\hat{\theta}(\mathbf{k}, t) - \frac{1}{\rho C_p}\int_0^\infty \widehat{\nabla\cdot\mathbf{q}_{r\nu}}\,\mathrm{d}\nu \qquad \text{(A3)}$$

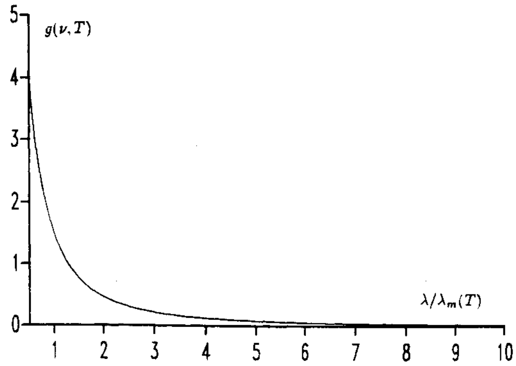

Fig. A1 Effects of the first neglected nonlinear term in the decomposition of $\delta I_\nu^b(T + \theta)$.

with

$$\widehat{\nabla\cdot\mathbf{q}_{r\nu}}(\mathbf{k}, t) = -4\pi\left(\tau_\nu'(0) + \int_0^\infty \frac{\partial^2\tau_\nu}{\partial r^2}(r)\frac{\sin(kr)}{kr}\,\mathrm{d}r\right) \times \left[\frac{\partial I_\nu^b}{\partial T}(T)\hat{\theta}(\mathbf{k}, t) + \frac{1}{2}\frac{\partial^2 I_\nu^b}{\partial T^2}(T)\hat{\theta}\otimes\hat{\theta}(\mathbf{k}, t)\right] \qquad \text{(A4)}$$

The convolution product $\hat{\theta}\otimes\hat{\theta}(\mathbf{k}, t)$ which appears in this equation is related to the nonlinear transfer between modes at different wave numbers. The resulting term in the equation for temperature spectrum needs further modeling.

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